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## Energy Dissipation Downstream of Labyrinth Weirs

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**Abstract:** *In order to evaluate the flow characteristics downstream of a rectangular labyrinth weir including energy dissipation an experimental study was carried out. The objective was to develop a relation between drop height, discharge and the amount of dissipated energy. For the physical tests a 0.60 m wide flume with a length of 18.00 m and a height of 1.20 m was used. For the design of the stilling basin combinations of discharges and downstream water levels were analyzed. The results show that for the given conditions a simple concrete apron or a riprap protection corresponding to the hydraulic jump length is sufficient for erodible river beds. For low downstream water levels the energy dissipation occurs in a steady hydraulic jump closed to the weir. Raising the downstream water level, the hydraulic jump becomes submerged and little energy will be dissipated. Due to its geometrical shape, the labyrinth weir can ensure overall an effective energy dissipation.*

**Keywords:** *Energy dissipation, labyrinth weir, fixed weir, waterway.*

### 1. Introduction

More than 300 weirs are operated in the federal waterway network in Germany. Most of them are movable weirs causing considerable operating and maintenance costs. To reduce these costs, fixed weirs represent an alternative to movable weirs. However, the requirements for the water level control must be satisfied. In addition to side weirs, labyrinth weirs could be an alternative at sites where the space is limited, the requirements for navigation are lower, and the discharge variation is small. Labyrinth weirs are an effective method to increase the discharge capacity because the discharge is proportional to the overflow length for free flow conditions. These weir types are characterized by a folded crest in plan view. Due to increased design floods, numerous spillways around the world have been already upgraded with these weir types. In the last couple of years there has been a significant amount of research on labyrinth weirs with respect to the hydraulic design. So far, there is only a little experience regarding the energy dissipation downstream of these weirs because most of the existing projects are equipped with large spillways. In comparison to a linear weir, it is assumed that more energy can be dissipated due to the folded shape leading to an interaction of the crossing nappes. Furthermore, local submergence effects in the upstream apex can be observed, which could also influence the energy dissipation.

#### 1.1. Labyrinth Weirs

The labyrinth weir is the simplest form of a weir with a folded crest. The geometry is based on vertical walls and allows an easy construction. Several design cycle options exist, such as rectangular, triangular, or trapezoidal. In the last couple of years, model studies and experimental research on labyrinth weirs increased with regard to the hydraulic capacity by varying geometrical parameters. A hydraulic and geometrical optimization is the Piano Key Weir (PKW), presented by Lempérière and Ouamane (2003). In comparison to labyrinth weirs, the upstream and downstream walls are inclined and PKW are suitable in particular at sites where the available space is limited, for example on top of gravity dams. Intensive research on labyrinth and PKW in the last years gives an overview about the improvement of the hydraulic design to increase the discharge capacity (Said and Ouamane 2011). Pralong et al. (2011) introduced a naming convention for PKW, which can be transferred to labyrinth weirs and is commonly used in the literature related to these weir types.

#### 1.2. Energy Dissipation

Until now, labyrinth and PKW are used mainly to increase the capacity of existing spillways. But, more and more folded weirs are installed in rivers. With regard to energy dissipation there is some literature on this subject mainly in combination with spillways. For example, stepped spillways, as explained in Silvestri et al. (2013), are a good possibility by providing natural aeration and by creating a skimming flow regime. In spite of the intensive research on the hydraulic efficiency of these weirs in the last couple of years, only a few studies have focused on the flow

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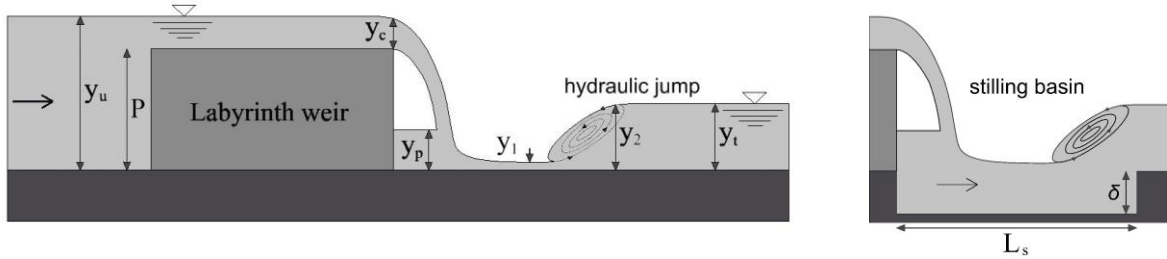
characteristics and energy dissipation downstream of labyrinth weirs in a horizontal channel. In Lopes et al. (2011) trapezoidal labyrinth weir models with different sidewall angles were tested. The results show a three-dimensional flow regime downstream of the labyrinth weir combined with air entrainment and shockwaves up to a certain discharge. Two-dimensional flow occurred further downstream and was evaluated as a function of the relation between the total upstream head  $H$  [m] over the weir crest and the weir height  $P$  [m] and the magnification ratio ( $L/W$ ) by measuring the downstream water depth at different points in the flow direction. Here  $L$  [m] corresponds to the crest length and  $W$  [m] corresponds to the clear width of the labyrinth weir. In Truong Chi and Ho Ta Khanh (2017) a PKW model on a dam crest with stepped spillway was tested with a stilling basin and a rockfill layer. The results show that a stilling basin can limit the downstream scour depth. Furthermore, with respect to scour danger, in Pfister et al. (2017) several tests on a PKW model with different sediment granulometry placed downstream of the PKW have been performed. The results show that the scour depth is higher for fine sediment in comparison to coarse sediment. Thus, it was recommended to protect the PKW from scouring by a plunge pool with a rough layer of riprap.

### 1.3. Aim of the Study

Lopes et al. (2011) analyzed the flow over a trapezoidal labyrinth weir, but studies at rectangular labyrinth weirs for varying tailwater flow conditions are missing. The question that arises is whether, without regard to scour danger, a stilling basin is needed and which form of hydraulic jump occurs for specific discharges and downstream water levels. Hence, the main purpose of this study is to test different configurations of stilling basins and to optimize the depth and length. Therefore, the performance of a rectangular labyrinth weir is compared under various flow conditions. Furthermore, the energy dissipation downstream of a labyrinth weir is compared to well-known formulas to find out if these approaches over- or underestimate the energy dissipation.

## 2. Theoretical Background

Energy losses at overflow weirs cause a higher upstream total head compared to the tailwater. During weir overflow the upstream head will be transformed to high velocities. A supercritical section with a flow depth  $y_1$  [m] less than critical depth  $y_c$  [m] behind the weir overflow can occur. The critical depth can be calculated by  $y_c = (q^2/g)^{1/3}$ . Further downstream the conjugated water depth  $y_2$  is greater than the critical depth. A hydraulic jump occurs and the flow becomes subcritical.



**Figure 1.** Definition sketch, flow at a labyrinth weir without (left) and with stilling basin (right).

The transition from supercritical to subcritical flow in a horizontal rectangular channel occurs in the form of a hydraulic jump depending on the Froude number  $Fr_1$  [-], the flow depth  $y_1$  [m], and the downstream depth  $y_2$  [m] (Chow 1959). To ensure safe energy dissipation downstream of the labyrinth weir, a stilling basin has to be designed. Therefore, two approaches can be used: first, the energy equation by Bernoulli without energy loss during overflow, and second, the approach by White which includes unknown energy losses in the formula (Naudascher 1987). For both the flow depth  $y_1$  [m] can be calculated.

Main parameters are the depth  $\delta$  [m] and length  $L_s$  [m] of the stilling basin. Further components like chute blocks, sills, or baffle piers were not considered. To protect the downstream bed and banks from scouring, the hydraulic jump should remain in the stilling basin. The tailwater ratio  $\varepsilon$  [-] is defined as follows:

$$\varepsilon = \frac{y_t + \delta}{y_2} \quad (1)$$

For  $\varepsilon > 1$  the hydraulic jump should remain in the stilling basin. For a safe design, the submergence ratio should be in the range  $1.05 < \varepsilon < 1.25$  (Strobl and Zunic 2006). For  $\varepsilon < 1$  the hydraulic jump moves downstream and for  $\varepsilon \gg 1$  the surface roller will be forced upstream.

An important parameter is the depth  $\delta$  of the stilling basin. In the range  $1.05 < \varepsilon < 1.25$ , the depth  $\delta$  can be expressed by Eq. (2).

$$\delta = (1.05 \div 1.25) \times y_2 - y_t \quad (2)$$

Another important parameter is the length of the stilling basin which corresponds often to the length of the hydraulic jump. In Zanke (2002) the length  $L_s$  [m] is approximated by the Eq. (2) of Smetana.

$$L_s \approx 6 \times (y_2 - y_1). \quad (3)$$

The United States Bureau of Reclamation (USBR) already developed standardized stilling basins on the basis of numerous model studies. The type of the stilling basin mainly depends on the Froude number  $Fr_I$  at the inlet of the stilling basin.

As already mentioned, the approach of White discussed in Naudascher (1987) is an option to determine the inflow water depth  $y_I$  in order to pre-design the stilling basin. It is based on the consideration that the nappe hits the water surface and a part of the kinetic energy dissipates. Based on the energy and momentum equation, White (Naudascher 1987) derived the following equation:

$$\frac{y_1}{y_c} = \frac{\sqrt{2}}{1.06 + \sqrt{\frac{\Delta z}{y_c} + \frac{3}{2}}} \quad (4)$$

where  $\Delta z$  [m] is the height of the free flow (at this point the height  $\Delta z$  corresponds to the weir height  $P$ ).

### 3. Experimental Setup

#### 3.1. Physical Model

Experimental tests were performed in a 0.60 m wide flume with a length of 18.00 m and a height of 1.20 m (Figure 2, left). The measurement of the water levels was carried out with two manual water level probes in the upstream and downstream channel. The discharge in the flume was controlled by a magnetic-inductive flowmeter (MID) and electrically adjustable valves. The channel is supplied by three pumps with a maximum discharge up to 420 l/s. The upstream entry of the channel has a grid of honeycombs to improve uniform flow conditions in the upstream water. The channel sides are made of glass to allow the observation of the flow patterns. The downstream water level was adjusted by a rectangular outlet flap. The upstream water level  $y_u$  [m] was measured 3.86 m upstream of the weir, and the downstream water level  $y_t$  [m] was measured 4.14 m downstream of the weir.

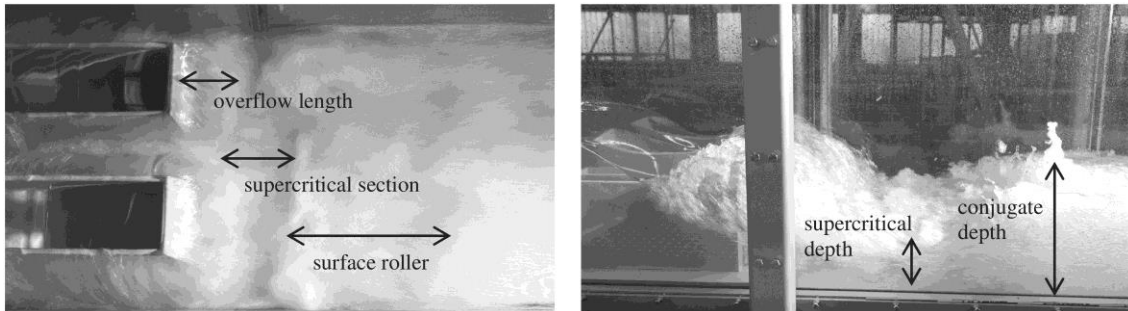


**Figure 2.** Flume for the physical model tests (left) and tested model (right) for  $q = 0.04 \text{ m}^3/(\text{s}\cdot\text{m})$ .

The tested rectangular labyrinth weir model (Figure 2, right) was made of  $T_s = 0.01 \text{ m}$  thick walls, creating the following geometry: weir height  $P = 0.255 \text{ m}$ , clear width  $W = 0.60 \text{ m}$ , inlet key width  $W_i = W_o = 0.125 \text{ m}$ , and two partial outlet keys with  $W_o = 0.110 \text{ m}$ , up- to downstream weir length  $B = 0.50 \text{ m}$ , and an overall crest length of  $L = 2.60 \text{ m}$  (with a ratio of  $n = L/W = 4.33$ ). Cicero and Delisle (2013) investigated the influence of different crest geometries on the discharge capacity and recommended a half rounded or quarter rounded crest for small heads up to  $H_u/P < 0.3$ , where  $H_u$  describes the total energy head in the upstream water during free flow conditions. This recommendation was adopted for the experiments by using a half rounded crest.

### 3.2. Test Series

Tests with specific discharges between  $0.02 \text{ m}^2\text{s}^{-1}$  and  $0.30 \text{ m}^2\text{s}^{-1}$  and downstream water levels between  $0.05 \text{ m}$  and  $0.30 \text{ m}$  were performed. For each combination of discharge and downstream water level, the appearance and the location of the hydraulic jump were visualized. With the help of a ruler the size of the nappe, the depth and length of the supercritical section behind the weir, the conjugate depth, and the length of the surface roller were measured (Figure 3). Due to the highly turbulent flow regime downstream of the weir, the measurement was carried out outside of the flume through the glass walls. In the case of an undular jump the conjugate depth was equal to the maximum crest wave. The extent of the first wave peak in the flow direction was defined as the length of the surface roller.

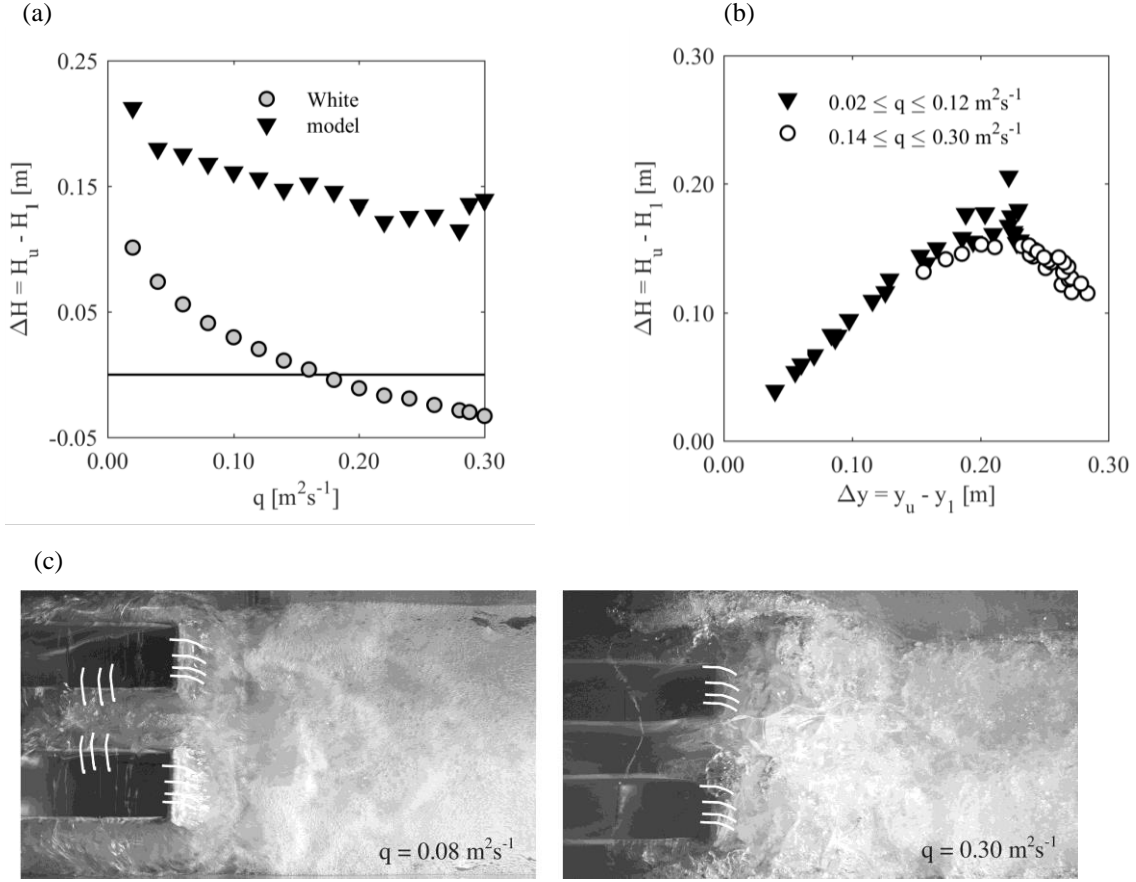


**Figure 3.** Measuring ranges.

## 4. Experimental Observations, Results, and Discussion

### 4.1. Energy Dissipation during Weir Overflow

Figure 4a shows a comparison of the calculated head difference  $H_u - H_1$  of White and the measured differences calculated from the experimental results. It can be confirmed that the three-dimensional flow over the labyrinth weir causes higher energy dissipation in comparison to the approach of White.



**Figure 4.** Deviation  $\Delta H$  between upstream energy head  $H_u$  and energy head  $H_l$  immediately behind the weir; lateral and longitudinal flow.

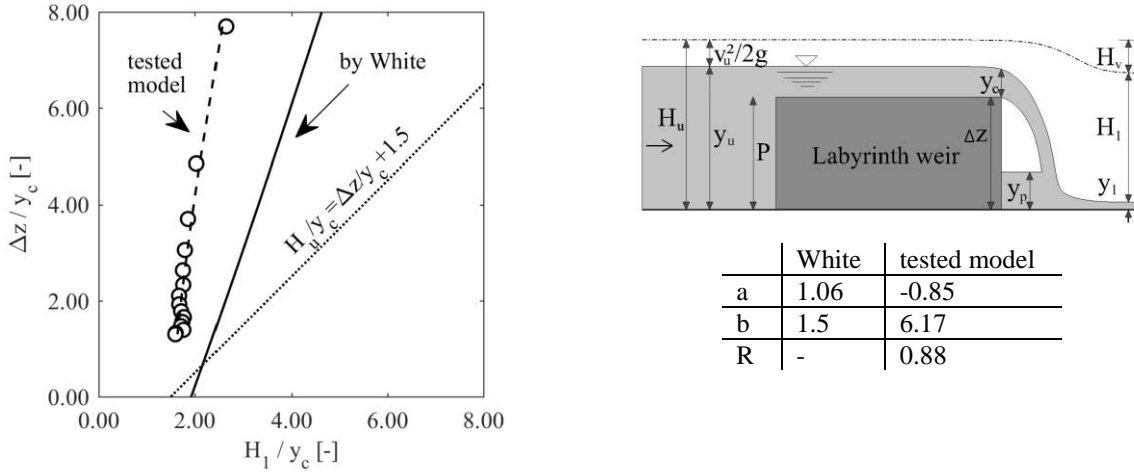
For our configurations, the approach of White is not appropriate for discharges higher than 0.18 m<sup>2</sup>s<sup>-1</sup> because this would result in higher energy heads downstream of the weir in comparison to the upstream water. Figure 4b shows the head difference  $\Delta H$  in comparison to the piezometric head difference  $\Delta y$ . For small drop heights  $\Delta y$  the energy dissipation is small and increases with rising head differences. This can be explained by the overflow characteristic of the labyrinth weir (Figure 4c). For small discharges the energy dissipation also takes place in the outlet key section and the nappes are aligned partially laterally and partially in flow direction. For higher discharges the water level in the keys increases due to local submergence effects (Crookston and Tullis 2011), and the nappes are only aligned in flow direction. Thus the energy dissipation decreases or remains almost constant. This effect can be observed in the model for unit discharges of  $q > 0.14$  m<sup>2</sup>s<sup>-1</sup> and is denoted in Figure 4b by black triangles and white circles, respectively.

#### 4.2. Comparison to the Approach by White

In Figure 5 the experimental results show that the energy dissipation at a labyrinth weir is higher in comparison to the approach of White. To be able to make statements about the energy losses in Naudascher (1987), Eq. (4) has been extended to the energy head  $H_l$  behind the nappe.

$$\frac{H_1}{y_c} = \frac{\sqrt{2}}{a + \sqrt{\frac{\Delta z}{y_c} + b}} + \frac{\left(a + \sqrt{\frac{\Delta z}{y_c} + b}\right)^2}{4}. \quad (5)$$

In the approach of White the coefficients are  $a = 1.06$  and  $b = 1.5$ . The corresponding curve (black line) can be seen in Figure 5. With increasing discharge, the energy head behind the nappe increased. Hence, the energy losses increased, which can be seen by the difference between the solid and dotted line, while the dotted line gets information about the energy head in the upstream water. As a result it can be shown for the tested labyrinth weir that the values (points) are on the left side of the curve by White. In comparison to a linear weir, the energy losses during the overflow over a labyrinth weir take place more strongly. For the tested labyrinth weir, the coefficients  $a$  and  $b$  in Eq. (5) could be determined. The coefficient of determination and the resulting curve are shown in Figure 5 as a dashed line.



**Figure 5.** Energy losses for the tested labyrinth weir model in comparison to the approach of White.

#### 4.3. First Test Series—Classification of the Hydraulic Jump

The experiments include two test series. First, tests were carried out with a rectangular labyrinth weir without stilling basin to get information about the flow characteristics in the downstream water. During the tests four manifestations of hydraulic jump could be observed, which are also described in Naudascher (1987):

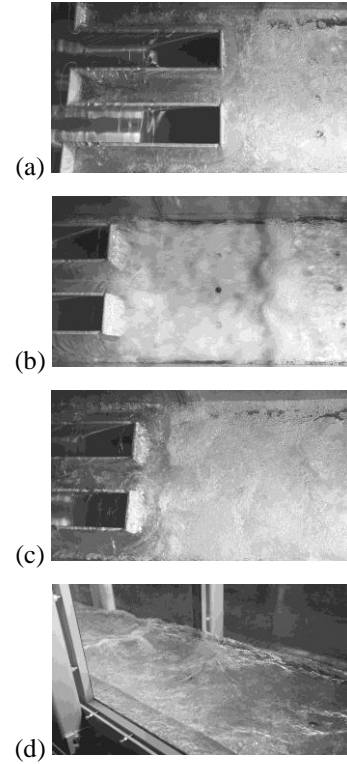
- No hydraulic jump with subcritical flow
- Hydraulic jump with a distinct surface roller
- Submerged hydraulic jump
- Undular jump; standing waves in the downstream water.

Table 1 shows the classification of the hydraulic jump for the investigated specific discharges and downstream water levels. For all states no drifted hydraulic jump could be observed and the energy dissipation always took place near the weir. For low downstream water levels and high discharges an undular jump with standing waves in the downstream water occurred. With increasing downstream water levels a hydraulic jump appeared in the form of a distinct surface roller or was completely submerged. For a specific discharge of  $0.02 \text{ m}^2\text{s}^{-1}$  and downstream water levels between  $0.10 < y_1 < 0.30 \text{ m}$  as well as for a downstream water level of  $0.30 \text{ m}$  and discharges between  $0.02 < q < 0.12 \text{ m}^2\text{s}^{-1}$ , no hydraulic jump could be observed.

For downstream boundary conditions with supercritical flow, as it can be found in rivers with a large slope, no flow transition occurred and consequently no stilling basin is required. Hence, there are no critical states with respect to the energy dissipation in the test series. As Lopes et al. (2011) already discussed, the flow over a labyrinth weir is basically three-dimensional. Both, the influence of the nappe and air entrainment favor a high energy dissipation during weir overflow. For the tested combinations of discharge, geometry, and drop height, no stilling basin is needed. If larger relevant discharges are included in future investigations, this statement is possibly no longer valid. However, as an additional safety, a stilling basin is designed for the present tested series.

**Table 1.** Classification of the hydraulic jump.

$y_t$ [m] \ $q$ [m <sup>2</sup> s <sup>-1</sup> ]	0.05	0.10	0.15	0.20	0.25	0.30
0.02	(b)	(a)	(a)	(a)	(a)	(a)
0.04	(b)	(b)	(c)	(c)	(c)	(a)
0.06	(d)	(b)	(b)	(b)	(c)	(a)
0.08	(d)	(d)	(b)	(b)	(c)	(a)
0.10	(d)	(d)	(b)	(b)	(c)	(a)
0.12	(d)	(d)	(b)	(b)	(c)	(a)
0.14	(d)	(d)	(d)	(b)	(b)	(c)
0.16	(d)	(d)	(d)	(b)	(b)	(c)
0.18	(d)	(d)	(d)	(b)	(b)	(c)
0.20	(d)	(d)	(d)	(d)	(b)	(b)
0.22	(d)	(d)	(d)	(d)	(b)	(b)
0.24	(d)	(d)	(d)	(b)	(b)	(b)
0.26	(d)	(d)	(d)	(d)	(b)	(b)
0.28	(d)	(d)	(d)	(d)	(b)	(b)
0.30	(d)	(d)	(d)	(d)	(b)	(b)

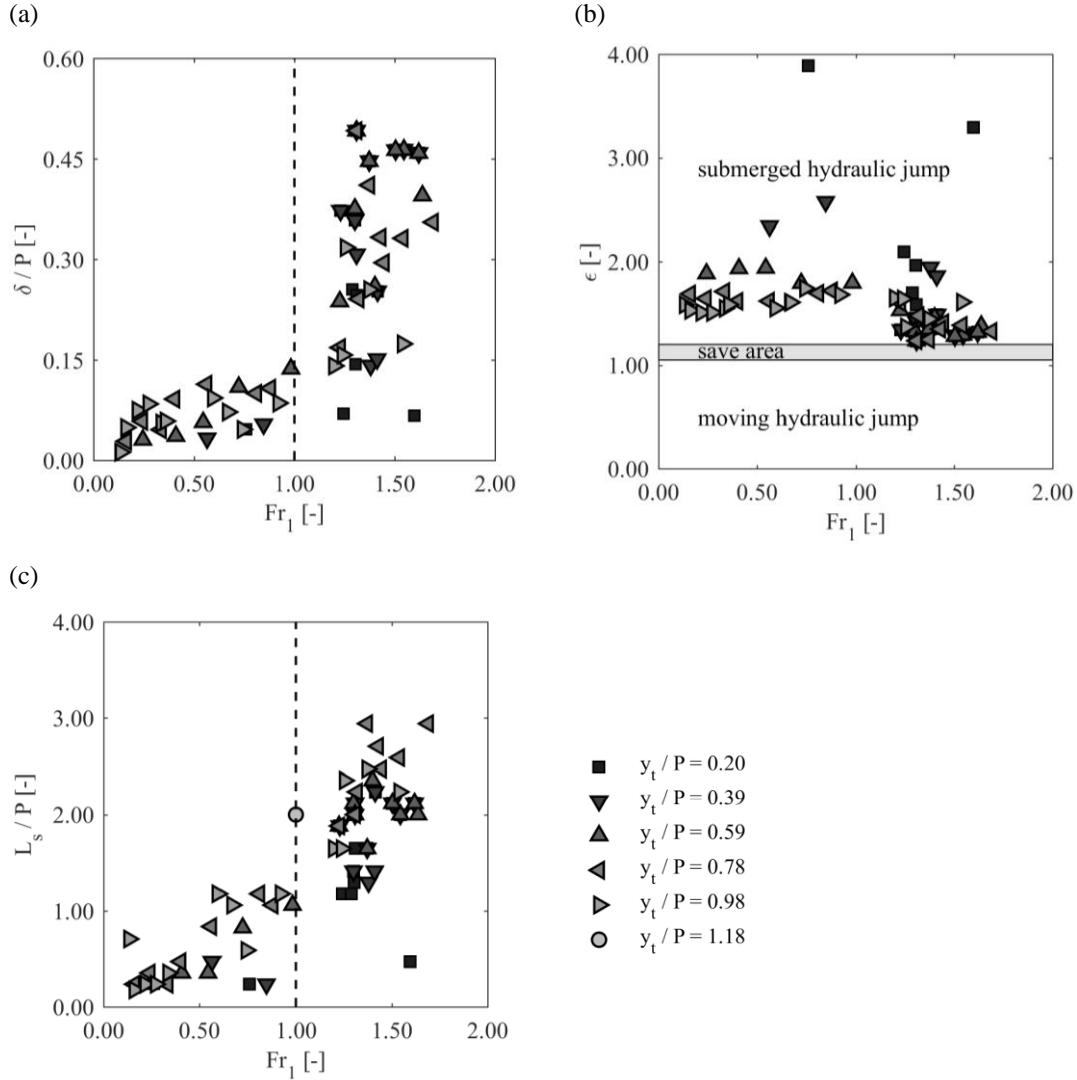


#### 4.4. Preliminary Design of the Stilling Basin

Due to the complex flow situation, empirical formulas are often used for the design of the stilling basins. As already mentioned, several standard designs have been developed by the USBR including baffles, end sills, and chute blocks. The function of chute blocks is to furrow the incoming jet and lift a portion of it from the floor (Chow 1959). However, these fixtures are controversial from an environmental point of view and will not be considered further in the subsequent investigations. The German directive DIN 19661-2 proposes four basic designs for the end sill of a stilling basin. They can be either vertical, sloped, or a combination.

In the present study the highest discharge in combination with the smallest downstream water level is the worst case for the depth of the stilling basin. According to Eq. (2) with a tailwater ratio of  $\varepsilon = 1.25$  a stilling basin of 0.13 m depth would be needed, neglecting that no stilling basin is needed like shown before (Figure 6a). This equals 50 % of the weir height. For this evaluation the depth would ensure a safe hydraulic jump with a tailwater ratio  $\varepsilon > 1$  (Figure 6b). There are several approaches in the literature for determining the length of the hydraulic jump. Empirical approaches by Smetana, Woycicki, and Tschertousow have in common that the length of a hydraulic jump is a function of the water depth  $y_1$ , the conjugated depth  $y_2$ , and the Froude number  $Fr_1$ . However, comparing the existing formulas, the differences for the lengths are remarkable, which may be explained by the difficulty to determine the length of the hydraulic jump. According to Hack (2009), the approach of Smetana is a good average. Calculating the length by Eq. (3) a maximum length of about three times of the weir height could be determined (Figure 6c).



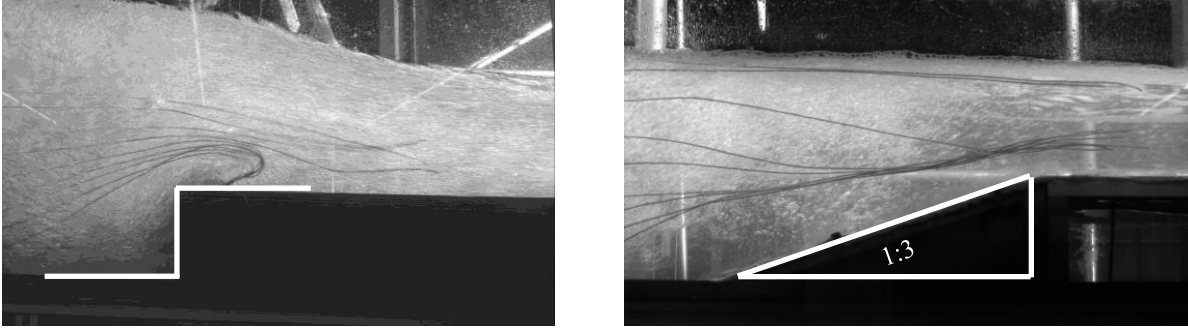


**Figure 6.** (a) Depth  $\delta$  of the stilling basin by Eq. (2); (b) Tailwater ratio  $\epsilon$  by Eq. (1); (c) Length  $L_s$  of the stilling basin by Eq. (3).

#### 4.5. Second Test Series—Studies with Stilling Basin

For the second test series experiments with a labyrinth weir with different stilling basin geometries were conducted. Here, the length of the stilling basin ( $L_s = 0.75, 0.80, 0.90, 1.30$  m) and the shape of the end sill (vertical and sloped) were varied. In particular, situations were examined which were classified as critical due to the occurrence of an undular jump. First, a stilling basin with a depth of 0.13 m, a length of 0.75 m, and a vertical sill was constructed. For all tested series, a submerged hydraulic jump was observed in the stilling basin. However, as a result of the vertical sill, a second flow transition occurs. During the test, a decrease of the water level downstream of the sill occurred, and for high discharges and high downstream water levels, a second hydraulic jump could be observed. Investigations with basin length of 0.80 m, 0.90 m, and 1.30 m and with a depth of  $\delta = 0.13$  m showed that the appearance of a second flow transition, regardless of its length, occurred with the same dimensions and flow depth. With the help of wool threads (Figure 7, left), it could be shown that the flow separated at the top of the sill. As a result, turbulent fluidized areas arose below the main flow which should be avoided in nature.

Therefore, the sill was ramped with an inclination of 1:3. This value was recommended also by Hack (2009), which allows a gradual transition from the bottom of the stilling basin to the downstream river bed.



**Figure 7.** Flow event in a stilling basin with a vertical sill (left) and a ramped sill (right) represented by wool threads.

First, the tests were carried out with a stilling basin with a length of 0.75 m and a depth of 0.13 m. With the sloped sill the second flow transition could be avoided for all considered states. Additionally, no flow separations occurred (Figure 7, right). Nevertheless, for high discharges a rough water surface in the stilling basin and in the downstream water could be observed. This can be explained by the increasing trajectory length of the nappes which hit the ramp area. Hence, the length of the stilling basin was extended to  $L_s = 0.90$  m. During the investigations, a significantly calmer water surface for the tested specific discharge could be noticed.

## 5. Conclusion

The aim of the investigation was to quantify the energy dissipation of a rectangular folded labyrinth weir. Therefore, experiments were conducted with regard to the danger of scouring in the downstream river bed. The test series showed that the energy dissipation at a labyrinth weir differs from other weir types due to its geometry. Because of the interaction between the lateral nappes and the resulting three-dimensional flow, air entrainment and shockwaves increase the energy dissipation. For the tested specific discharges and downstream water levels, no states could be identified for which a deep stilling basin would be necessary. For large downstream water levels a submerged hydraulic jump occurred. In the case of small specific discharges in combination with high heads, the energy dissipation during weir overflow caused a local hydraulic jump with a distinct surface roller. Nevertheless, the use of a plane stilling basin is recommended against erosion. Theoretical design equations overestimated the geometry of the stilling basin because the energy dissipation during weir overflow is neglected. As a result of the tests, for the investigated conditions, a stilling basin with  $L_s/P = 3.5$  and  $\delta/P = 0.5$  is recommended to ensure a safe hydraulic jump in the stilling basin. In addition, a sloped end sill with an inclination of 1:3 prevented another flow transition in the downstream water. Nevertheless, considering economic optimizations, further studies with smaller basin depths should be performed. Furthermore, it should be examined whether the recommended design parameters can be generalized beyond the investigated boundary conditions.

## 6. Acknowledgements

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